Tema PS – Ungureanu Matei

Variabile aleatoare discrete unidimensionale

DT = document scris de tipar DM = document scris de mana

Tema Bonus: Cele doua calcule de la seminar de la exercitiul 12 au fost facute cu formulele E(X) = n\*p si Var(X) = n\*p\*(1-p)

DT.2. Calculați media și dispersia următoarelor variabile aleatoare:

a)

CE: 6p+p+4p^2+2p+3p^2+3p^2=1, p>0

10p^2+9p-1=0

Delta = 81+4\*10=121=11^2 => p1=(-9+11)/20=0.1 >0 p2=(-9-11)/20 <0 => p = 0.1

0 1 2 3 4 5

X: (0.6 0.1 0.04 0.2 0.03 0.03)

E(X) = 0 + 0.1 + 0.08+ 0.6 + 0.12 + 0.15 = 1.05

Var(X) = E(X^2) – (E(X))^2 = 0 + 0.1 + 0.16 + 1.8 + 0.42 + 0.75 – (1.05)^2 = 2.1275

b)

E(X) = (Suma de la k=1 la infinit din) k\*P(X=k) = (Suma de la k=1 la infinit din) k \* p \* q^(k-1) = p / (1-q)^2 (pentru |q|<1)

E(X^2) = (Suma de la k=1 la infinit din) k^2 \* p \* q^(k-1) = p(1+q) / (1-q)^3

Var(X) = p(1+q) / (1-q)^3 – p^2 / (1-q)^4 = (p(1-q^2) – p^2) / (1-q)^4

c)

E(X) = (Suma de la k=0 la infinit din) k\*P(X=k) = (Suma de la k=0 la infinit din) k \* p \* (1/3)^k = p \* (1/3) / (1-1/3)^2 = 3p/4

E(X^2) = (Suma de la k=0 la infinit din) k^2 \* p \* (1/3)^k = p(1/3)(1+1/3) / (1-1/3)^3 = 3p/2

Var(X) = 3p/2 – 9p^2/16 = (24p – 9p^2) / 16

d)

E(X) = (Suma de la k=0 la infinit din) k \* P(X=k) = (Suma de la k=0 la infinit din) k \* p \* (1/2)^k = p \* (1/2) / (1-1/2)^2 = 2p

E(X^2) = (Suma de la k=0 la infinit din) k^2 \* p \* (1/2)^k = p(1/2)(1+1/2) / (1-1/2)^3 = 6p

Var(X) = 6p – 4p^2

DT.6.



X, subpunctul a, si subpunctul b sunt la fel ca la exercitiul 1, facut in clasa.

E(X) = -7/11

E(X^2) = 21/11

Var(X) = 182/121

F(X) = P(X<=x) = 0, x<-2

3/11, -2<=x<-1

7/11, -1<=x<0

9/11, 0<=x<1

10/11, 1<=x<2

11/11, 2<=x

c) Calculaţi E(3X − 2), Var(6X − 3), E(X + X^2)

E(3X-2) = 3E(X) – 2 = -21/11 – 2 = -43/11

Var(6X-3) = 36Var(X) = 6552/121

E(X + X^2) = E(X) + E(X^2) = -7/11 + 21/11 = 14/11

d) Calculaţi P(|X| < (1/2) / -1.25 < X < 0.75)

(presupun ca / reprezinta probabilitate conditionata, eu de obicei o notez cu |)

P(A/B) = P(A intersectat cu B) / P(B)

P((|X| < (1/2)) = P(-1/2 < X < 1/2) = 2/11

P(-1.25 < X < 0.75) = 4/11 + 2/11 = 6/11

P(|X| < (1/2) intersectat cu -1.25 < X < 0.75) = 2/11

P(|X| < (1/2) / -1.25 < X < 0.75) = (2/11) / (6/11) = 1/3

DT.11. Fie X variabila aleatoare ce indică numărul de puncte obținute la aruncarea

unui zar. Să se determine parametrii reali a și b astfel ȋncat momentul centrat de

ordin 2 al variabilei aleatoare Y = aX +b să fie egal cu 1.

Var(aX+b) = 1

1 2 3 4 5 6

X: (1/6 1/6 1/6 1/6 1/6 1/6)

pk = (Combinari de 6 luate cate k) \* p^k \* (1-p)^(6-k) k de la 1 la 6, p = 1/6

E(X) = (1+2+3+4+5+6) / 6 = 21/6

E(X^2) = (1+4+9+16+25+36) / 6 = 91/6

Var(X) = 91/6 – 441/36 = 105/36 = 35/12

a^2 \* 35/12 = 1 => a1 = radical(12/35), a2 = -radical(12/35)

DM.1. Pornind de la X si Y (v.a. discrete independente), construiti urmatoarele v.a.:

a)

6 9

3X: (1/5 4/5)

1/2 1/3

X^(-1): (1/5 4/5)

-1 0

cos(pi/2 \* X) : (1/5 4/5)

9 4

Y^2: (4/5 1/5)

0 1

Y+3 : (4/5 1/5)

b)

-1 8

X-1: (1/2 1/2)

0 1/81

X^(-2): (1/2 1/2)

0 radical(2)/2

sin(pi/4 \* X): (1/2 1/2)

-15 5

Y\*5: (1/7 6/7)

e^(-3) e

e^Y: (1/7 6/7)

c)

10 16

2X: (1/3 2/3)

1/125 1/512

X^(-3): (1/3 2/3)

0 0

tg(pi\*X): (1/3 2/3)

-3 -1

Y-2: (1/6 5/6)

1 1

|Y|: (1/6 5/6)

d)

5 -4

2-X: (1/8 7/8)

-27 216

X^3: (1/8 7/8)

0 -1

cos(pi/6 \*X): (1/8 7/8)

e^(-1) e^(-3)

Y^(-1): (1/4 3/4)

1 3

ln Y: (1/4 3/4)

DM.2. Folosind v.a. X si Y de la 1) determinati:

a)

-5 -2 -3 0

2X+3Y: (4/25 1/25 16/25 4/25)

9 8 12 11

3X-Y: (4/25 1/25 16/25 4/25)

-108 -32 -243 -72

X^2\*Y^3: (4/25 1/25 16/25 4/25)

b)

3 −1 12 8

X-Y: (1/14 3/7 1/14 3/7)

1 1 -1 -1

cos(pi\*X\*Y): (1/14 3/7 1/14 3/7)

-9 3 72 84

X^2+3Y: (1/14 3/7 1/14 3/7)

c)

4 6 7 9

X+Y: (1/18 5/18 1/9 5/9)

-1 1 0 0

sin(pi/2 \*X\*Y): (1/18 5/18 1/9 5/9)

-4/5 6/5 -7/8 9/8

1/X + 1/Y: (1/18 5/18 1/9 5/9)

d)

-3e -3e^3 6e 6e^3

X\*Y: (1/32 3/32 7/32 21/32)

-3/e -3/e^3 6/e 6/e^3

X/Y: (1/32 3/32 7/32 21/32)

3+e^2 3+e^6 |6-e^2| |6-e^6|

|X-Y^2|: (1/32 3/32 7/32 21/32)

DM.3. Determinati parametrii reali p si q stiind ca X si Y sunt v.a. bine definite:

a)

p+q = 1, p,q>0

* 1. + (p^2 + 0.02) / 2 = 1 => p^2 = 0.43 => p=radical(0.43), q = 1-p

b)

1/3 + p + q^2 = 1, p>0

p+p+p^2 = 1 => p^2 + 2p – 1 = 0

delta = 4+4=8 => p=(-2+radical(8))/2=-1+radical(2) (cea cu minus nu e buna pentru ca avem nevoie ca p sa fie intre 0 si 1 in cazul de fata)

q^2 = 1-p-1/3 = 2/3+1-radical(2) = (5-radical(18))/3 => q = (+/-)radical((5-radical(18))/3)

(banuiesc ca q nu e necesar sa fie mai mare ca 0, pentru ca in X apare q^2 oricum)

Nu inteleg o chestie la exercitiul asta totusi: daca avem v.a. X, si aflam X^2, atunci probabilitatile nu se schimba, doar valorile se ridica la patrat. Deci probabilitatile de la X trebuie sa fie respectiv egale cu cele de la X^2. Deci p ar fi 1/3, p^2 ar fi 1/9, si nu se mai respecta conditia de existenta, ca suma sa dea 1, nici la X nici la X^2. Am gresit eu undeva?

c)

p + p^2 + q = 1, p,q>0

1 0

X^4: (p+q p^2 q)

16/25 = p+q

9/25 = p^2 => p = 3/5 (p>0)

q = 16/25 – 15/25 = 1/25

d)

2p + q = 1 p,q>0

q + 7q = 1 => q=1/8

p = 7/16

DM.4. Folosind repartitiile v.a. de la 1),2) calculati:

(modul prin care am rezolvat P(AB conditie 1 |B conditie 2): rezolv conditia 2 , gasesc valorile lui B care merg, si pentru fiecare din ele verific in prima expresie pentru ce valori ale lui A conditia 1 e indeplinita)

a)

P(2X+3Y > 1) = 0

P(2X+3Y > 1 | X>0) = 0

P(2X+3Y < 3 | Y<-2) = 1

P(X^2\*Y^3>3) = 0

P(X^2\*Y^3<=3) = 1

P(2X+3Y < 3X-Y) = 1

b)

P(X-Y>0) = 4/7

P(X−Y<0 ∣ X>0) = 0

P(X−Y>0 ∣ Y<=0) = 1

P(cos(pi\*X\*Y) < 1/2​) = ½

P(X2+3Y>=3) = 13/14

P(X-Y<X^2+3Y) = 13/14

c)

P(X+Y<2) = 0

P(X+Y>2 | X>5) = 1

P(X+Y<12 | Y<0) = 1

P(sin(pi/2 \*X\*Y) <= 1/2) = 0

P(1/X+1/Y<1 | Y<0) = 1

P(1/X+1/Y<X+Y) = 1

d)

P(X\*Y<=e^4) = 11/32

P(X\*Y>=7 | X<0) = 0

P(X\*Y<9 | Y>3) = 1/8

P(X/Y<1) = 25/32

P(|X-Y^2|>=3) = 25/32

P(X/Y<|X-Y^2|) = 25/32